Optimization of Spectrally and Spatially Flexible Optical Networks with Spatial Mode Conversion

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***Abstract*—Spectrally and spatially flexible optical networks (SS-FONs), which combine space division multiplexing (SDM) with flexible-grid elastic optical network (EON) technologies, bring additional complexity to network control due to the handling of a larger number of spatial modes in SDM than in conventional EONs. To effectively optimize such networks, in particular, to generate good-quality solutions of low optimality gaps in reasonable computation times, efficient algorithms are required. In this work, we focus on the routing, spatial mode, and spectrum allocation (RSSA) problem in the SS-FONs in which conversion of spatial modes in switching nodes is allowed. We propose and evaluate two enhancements in RSSA processing, namely, algorithm parallelization and application of dedicated data structures, which are built into a hybrid simulated annealing with greedy RSSA heuristic algorithm. To assess the quality of generated RSSA solutions, we develop suitable procedures for estimating solution lower bounds. The results of numerical experiments show the effectiveness of proposed techniques in speeding up the search for good-quality RSSA solutions.**

***Index Terms*—optical networks, space division multiplexing, routing, space and spectrum allocation, network optimization, offline planning, parallel algorithm, column generation**

I. Introduction

Space division multiplexing (SDM) is a forthcoming optical network technology going beyond the capabilities of fixed-grid wavelength division multiplexing (WDM) and flexible-grid elastic optical network (EON) systems by enabling parallel transmission of several co-propagating spatial modes in suitably designed optical fibers [1], [2]. SDM, when combined with multi-carrier (i.e., super-channel, abbreviated as SCh) and distance-adaptive transmission enabled by EONs, brings many benefits including enormous increase in transmission capacity, extended flexibility in resource management due to the introduction of the spatial domain, as well as potential cost savings thanks to the sharing of resources and the use of integrated devices [3]. The feasibility of the spectrally-spatially flexible optical network (SS-FON) concept has been demonstrated in [4].

The main concern in optical networking is provisioning of lightpath connections for transmitted signals. A lightpath is an optical path established between a pair of source-destination nodes. In SS-FONs, the lightpaths carrying SChs are routed through the network over the spatial modes of SDM suitable links within an appropriately assigned spectrum segment. Having a set of traffic demands, the routing of lightpaths requires a contention-free allocation of spectrum resources on spatial modes of each link belonging to the routing path of each connection realizing a demand. It translates into the problem of routing, spatial mode, and spectrum allocation (RSSA), which consists of finding lightpath connections, tailored to the actual width of transmitted signals (i.e., SChs), for end-to end demands that compete for spectrum and spatial resources [5]. The RSSA problem is present both in the phase of offline network planning and during its operation. The former case is more challenging since it involves the establishment of lightpath connections for a set of traffic demands, and such optimization problem is know to be $\mathcal{NP}$-hard [5]. The latter case concerns mainly the provisioning of lightpaths for connection requests that arrive and disappear stochastically (i.e., one-by-one). Even here, the complexity of RSSA may be high if in-operation planning (i.e., network re-optimization during its operation) is performed [6].

The handling of a much larger number of spatial modes than in single-mode EONs dramatically increases the complexity of both hardware and control functions in SDM networks. It results in a large set of decision variables in network optimization, which makes RSSA more complex than the routing and spectrum allocation (RSA) problem in EONs [7]. Consequently, efficient algorithms for solving the RSSA problem are required in such networks. In the following, we discuss related works and present our contributions.

A. Related Works

In network/connection planning, the decision how to allocate the traffic demands is made in an off-line manner, with relaxed processing time constraints (when compared to dynamic connection provisioning). Therefore, more complex and time consuming optimization methods can be applied for solving such decision problems. Among them, the most usual one considered for RSSA-related problems is the mixed-integer programming (MIP) modeling approach (see e.g., [8]). Also simple greedy algorithms are quite frequently used (e.g., [9]). The least popular are meteheuristics, which in most cases employ a simulated annealing (SA) approach (see e.g., [10]).

The advantage of MIP is that it yields exact (i.e., globally optimal) solutions. However, its key shortcoming is low scalability, i.e., it cannot provide optimal or even feasible results in a reasonable time for larger instances of complex problems, which is the case of most of optimization problems in optical networks. Contrarily, (meta-)heuristics can effectively generate feasible solutions to large-scale problems relatively quickly; however, they do not guarantee their optimality. Indeed, in

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many flexible-grid EON scenarios such methods may face scalability problems and have difficulties with providing solutions close to optimal ones [11].

One way to speed up the generation of good-quality solutions is to employ algorithm parallelization. The processing power of CPUs and GPUs in terms of the number of processing cores is continually increasing. It enables the possibility to run a number of threads, where each thread performs its own search for optimal solutions in the solution space. Effective application of GPUs in optimization of EONs has been demonstrated in [12]. To the best of our knowledge, algorithm parallelization in SS-FONs has not been studied yet.

Another way to increase the efficiency of optimization procedures is to use appropriate data structures. A basic representation of the allocation status of spectral-spatial resources is by means of a matrix structure, for instance, a matrix of binary variables, where each matrix element denotes whether given frequency slice on the corresponding spatial mode is used or not. In [13], another matrix representation that indicates the size of available contiguous spectrum blocks is proposed. Using this matrix, it is enough the check the value of a single matrix element so that to know if a demand can be accommodated in a given frequency slot, while a binary representation involves the iteration over a number of consecutive slices. As we show in this paper, further improvements can be achieved.

Since (meta-)heuristics do not have in-built optimality guarantees, additional procedures aiming at estimation of solution lower bounds (LBs) can be used for evaluation of solution quality. In particular, the lower the relative difference (referred to as *optimality gap*) between the values of generated feasible solution and LB the higher quality of the solution. Note that whenever these values equal (i.e., the optimality gap is 0%), then it is assured that the solution is (globally) optimal. In [14] and [15], the LBs are estimated in EON scenarios by solving an MIP model of the RSA problem in which the spectrum continuity constraint is relaxed. We have not found any similar works in the context of SS-FONs.

For more details on resource allocation schemes in SS-FONs as well as on optimization models and algorithms considered for the RSSA problem refer to our recent comprehensive literature survey presented in [5].

B. Assumptions and Contributions

We focus on an offline network planning problem, which concerns establishing lightpath connections for a set of traffic demands competing for spectral-spatial resources with a goal to optimize their utilization. The considered problem translates into an RSSA optimization problem in which the width of spectrum required in the network to allocate the demands is subject to minimization. As a case study scenario, we assume an SS-FON in which spectral SChs are carried by lightpaths over the spatial resources of optical links consisting of single-mode fiber bundles (SMFB). The lightpaths have assigned frequency slots that do not change on their routing paths (i.e., the spectrum continuity constraint is imposed). However, the network nodes allow for conversion of spatial modes, i.e., for lane changes and switching of modes between any input and output ports. Accordingly, different modes can be assigned to a lightpath in the links belonging to its routing path (i.e., the so-called spatial continuity constraint is relaxed). As discussed in [5], this is one of the most frequently considered scenarios in the literature in the context of SS-FONs.

Our main contribution is development of an efficient RSSA algorithm, based on a standard simulated annealing algorithm combined with a greedy RSSA heuristic, that introduces two enhancements in SS-FON optimization: parallel processing and improved data structures. We also develop suitable procedures for estimation of solution lower bounds, with the aim to evaluate the quality of generated RSSA solutions. As the obtained numerical results show, the proposed techniques significantly speed up the search for optimal RSSA solutions.

The rest of the paper is organized as follows. In Section II, we formulate the considered RSSA problem. In Section III, we describe the optimization algorithm. In Section IV, we present the procedures for estimation of lower bounds. In Section V, we present and discuss the results of numerical experiments. Eventually, we conclude the work in Section VI.

II. Problem Formulation

We formulate the RSSA problem as an MIP problem using the link-lightpath (LL) modelling approach [16].

The considered SS-FON is represented by graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V}$ is the set of optical nodes and $\mathcal{E}$ is the set of fiber links. The set of spatial modes available on each link is denoted as $\mathcal{M}$. On each mode $m \in \mathcal{M}$, for each network link $e \in \mathcal{E}$, the same bandwidth (i.e., optical frequency spectrum) is available and it is divided into set $\mathcal{S} = \{s_1, s_2, \ldots, s_{|\mathcal{S}|}\}$ of frequency slices of a fixed width. The set of (traffic) demands to be realized in the network is denoted by $\mathcal{D}$.

In the LL model, a notion of a lightpath is used. A lightpath is understood as tuple (p, c), where p is a route and c is a frequency slot. The route is a path through the network from the source node to the termination node of a demand ($p \subseteq \mathcal{E}$), while the frequency slot is a set of contiguous slices (the property called the spectrum contiguity constraint) assigned to the lightpath ($c \subseteq \mathcal{S}$). Frequency slot c should be wide enough to carry the bit-rate of demand d on path p, if it is supposed to satisfy this demand. Note that the width of c (i.e., $|c|$) may differ in the function of the length of path p. This fact allows us to model the distance-adaptive transmission, where the best possible modulation format is selected for each candidate path [11]. Frequency slot c is the same for each link belonging to the routing path (according to the spectrum continuity constraint). The set of allowable lightpaths for demand $d \in \mathcal{D}$ is denoted as $\mathcal{L}(d)$. Finally, let $\mathcal{L}$ be the set of all allowable lightpaths.

The RSSA problem in the considered SS-FON scenario with the spatial mode conversion simplifies to selecting one of the allowable lightpaths for each demand in such a way that the sum of lightpaths utilizing the same slice on the same link does not exceed the number of available spatial modes (as the spatial continuity constraint is relaxed). As a consequence, each lightpath is assigned a binary variable $x_{dl}, d \in \mathcal{D}, l \in \mathcal{L}(d)$, where $x_{dl} = 1$ indicates that lightpath l is actually set-up and it carries the traffic of demand d. Besides, each binary variable $y_{es}, e \in \mathcal{E}, s \in \mathcal{S}$, indicates if there is a used lightpath allocated on slice s of link e. Eventually, the use of slice s in the network is indicated by a binary variable $y_s, s \in \mathcal{S}$.

The corresponding MIP formulation of RSSA is as follows:

$$\text{minimize } z = \sum_{s\in\mathcal{S}} y_s \quad \text{(1a)}$$

$$[\lambda_d] \quad \sum_{l\in\mathcal{L}(d)} x_{dl} = 1 \qquad d \in \mathcal{D} \quad \text{(1b)}$$

$$[\pi_{es} \geq 0] \quad \sum_{l\in\mathcal{L}(e,s)} x_{d(l)l} \leq |\mathcal{M}| y_{es} \quad e \in \mathcal{E}, s \in \mathcal{S} \quad \text{(1c)}$$

$$\sum_{e\in\mathcal{E}} y_{es} \leq |\mathcal{E}| y_s \qquad s \in \mathcal{S}, \quad \text{(1d)}$$

where $\mathcal{L}(e, s)$ is the set of lightpaths routed through link e and slice s, and $d(l)$ is the demand realized by lightpath l. Optimization objective (1a) minimizes the number of the slices actually used (equal to the sum of variables y_s). Constraint (1b) assures that each demand will use exactly one lightpath from the set of allowable lightpaths. Constraint (1c) assures that there are no collisions of the assigned resources, i.e., the sum of lightpaths utilizing the same slice on the same link does not exceed the number of spatial modes. Finally, constraint (1d) defines variables y_s that indicate whether slice s is used on at least one link.

Note that the solution of (1) does not provide explicit information about the spatial modes utilized on consecutive links of the selected lightpaths. Therefore, to obtain a feasible assignment of modes to the lightpaths, a simple post-processing procedure presented in [8] can be applied.

Solving model (1) is difficult even in the case of EONs (i.e., for $|\mathcal{M}| = 1$) [15]. However, the linear relaxation of (1) (referred to as LP) can be useful in estimation of the solution lower bounds, for which we develop a suitable column generation-based procedure in Section IV-B. The procedure makes use of the dual variables associated with the respective constraints of LP, which are denoted in (1) by symbols λ_d and π_{es}.

III. Generating RSSA Solutions

In the search of optimal solutions for the RSSA problem formulated in Section II, we apply a similar optimization approach as in [17], which is a combination of greedy lightpath allocation (GLA) and simulated annealing (SA) – in this paper, we denote it as an SA-GLA algorithm. In particular, GLA processes demands one-by-one, according to a given demand order, and assigns to them lightpaths in such a way that each assignment minimizes cost function (1a) (i.e., spectrum usage). Here, the best routing path from the set of allowable paths $\mathcal{P}$ and vector of spatial modes is selected for each demand, whereas spectrum is allocated using a first-fit (FF) policy. The width of allocated frequency slot is calculated assuming the most spectrally efficient modulation format, but such that its transmission reach exceeds the path length. The demand order is being optimized iteratively by applying SA, in a similar way as in [18]. In particular, at each iteration, SA swaps the order of just two randomly selected demands and, for such new order, it calls the GLA procedure. If the new order leads to the improvement in the objective function, it is considered as the best one and accepted as the current order in next iterations. Otherwise, it is accepted as the current one with certain probability that decreases during SA processing. In the SA-GLA algorithm, GLA is capable of producing feasible RSSA solutions quickly, while SA explores the feasible solution space in the search for (locally) optimal solutions.

The time required to generate optimized RSSA solutions increases with the number of spatial modes and it can be considerable. Indeed, as reported in [7], algorithm processing times might be of the order of tens or hundreds of seconds even when using a simple greedy heuristic approach in SS-FONs supporting spectral-spatial SChs. Therefore, to speed-up the search for optimal RSSA solutions, we propose two enhancements in the SA-GLA algorithm processing, namely, parallelization of SA and application of suitable data structures for efficient search of free spectrum resources in GLA.

A. Parallel Simulated Annealing

In this work, we implement a basic approach for parallelization of our optimization algorithm, in which a number of parallel threads is run on a multi-core CPU, where each thread is associated with a logical CPU core, and each thread calls its own, independent instance of SA-GLA. The instances of SA-GLA are initialized with different orders of demands that, after calling the GLA procedure, result in different initial RSSA solutions. They perform random and uncorrelated swaps of pairs of demands and explore the solution space in the search for optimal RSSA solutions without any exchange of information about their current best solutions. After meeting the termination condition, which in our implementation happens either when the objective value of found solution equals the solution lower bound (estimated in a pre-processing phase) or given computation time limit is exceeded, the best solution found among all the threads is returned.

B. Efficient Spectrum Search

The GLA procedure aims at selecting the best lightpath (i.e., such that minimizes cost function (1a)) for each consecutively processed demand $d \in \mathcal{D}$. It checks iteratively all allowable paths from set $\mathcal{P}(d)$ and looks for a free frequency slot, the same on all links belonging to the path (due to the spectrum continuity constraint), on any spatial mode in each link of the path (since the space continuity constraint is relaxed). GLA applies the FF spectrum allocation policy. Namely, the allocation status of frequency slices in set $\mathcal{S}$ is checked starting from the lowest indexed slice (i.e., slice s_1) until the required number of free consecutive slices (denoted as N) that form the frequency slot is found on any spatial mode in each link of the path. If found, the frequency slot on the the lowest-indexed spatial mode (among possible ones) is selected.

The allocation status of spectral-spatial resources in a network link can be represented in a form of matrix $\mathbf{A}^{|\mathcal{M}|\times|\mathcal{S}|}$. Below, we present four alternative ways in which this data structure can be defined and processed.

1) **Slice Allocation Status (SAS)** – a basic approach used, e.g., in [15], in which $\mathbf{A} = (a_{ms}) \in \{0, 1\}^{|\mathcal{M}|\times|\mathcal{S}|}$, where element $a_{ms} = 0$ indicates that slice s on mode m is free, and $a_{ms} = 1$ otherwise; see Fig. 1(a). In SAS, the

frequency slots in the order: $(s_1, ..., s_N)$, $(s_2, ..., s_{N+1})$, $(s_3, ..., s_{N+2})$, etc., are checked until the first one having all the slices unoccupied for some $m \in \mathcal{M}$ is found. Note that SAS involves the iteration over a number of slices to check the availability of a frequency slot.

2) **Maximal Free Block (MFB)** – a data structure proposed in [13], in which $\mathbf{A} = (a_{ms}) \in \mathbb{N}^{|\mathcal{M}|\times|\mathcal{S}|}$, where the (non-negative integer) value of element a_{ms} indicates the size of available contiguous spectrum block beginning from slice s on mode m; see Fig. 1(b). Similarly as in SAS, the scan of spectrum is performed for consecutive beginning frequency slices: $s_1, s_2, s_3, ...$, however, it is enough to check and satisfy the condition: $a_{ms} \geq N$ for certain $m \in \mathcal{M}$, so that to find the required frequency slot.

3) **Maximal Free-Occupied Block (MFOB)** – the approach that we propose in this paper, in which $\mathbf{A} = (a_{ms}) \in \mathbb{Z}^{|\mathcal{M}|\times|\mathcal{S}|}$, where the positive (integer) value of element a_{ms} indicates the size of available contiguous spectrum block beginning from slice s on mode m (similarly to MFB), and its negative value indicates the size of occupied contiguous spectrum block; see Fig. 1(c). As above, the search for a free frequency slot starts from beginning slice s_1 but, on the contrary to MFB, if $a_{ms_1} < N$, then the next one to be checked is at position $s_1 + |a_{ms_1}|$, and so on. In this way, the intermediate slices are skipped from processing since they obviously does not provide enough free resources to establish the required frequency slot.

4) **MFOB with Aggregation (MFOB-A)** – in this extended version of MFOB, which is suitable for SS-FONs with spatial mode conversion, auxiliary vectors $\mathbf{b}$ and $\mathbf{c}$ are used. Vector $\mathbf{b}$ is defined as $\mathbf{b} = (b_1, ..., b_{|\mathcal{S}|}) \in \mathbb{N}^{|\mathcal{S}|}$, where $b_s = max\{a_{ms} : m \in \mathcal{M}\}$ and it indicates the size of the largest free spectrum block beginning from slice s among all spatial modes. Vector $\mathbf{c}$ is defined as $\mathbf{c} = (c_1, ..., c_{|\mathcal{S}|}) \in \mathbb{N}^{|\mathcal{S}|}$, where $c_s = min\{|a_{ms}| : m \in \mathcal{M}\}$ and it indicates the size of the smallest spectrum block (either free or occupied) among all spatial modes that begins from slice s. The condition that terminates the search for a free frequency slot is: $b_s \geq N$, since it is assured that on at least one spatial mode there is such slot available that begins from slice s. If this condition is not met, the next check for free spectrum resources is performed at slice position $s + c_s$.

Note that both MFOB and MFOB-A have some processing overhead due to necessary updates of data structures after each lightpath allocation. Still, the overall benefits from accelerated spectrum search considerably outweigh this drawback.

IV. Estimating Lower Bounds

In this section, we develop two alternative methods for estimating the LBs of the RSSA problem formulated in Section II, one making use of the relaxed MIP formulation (referred to as LB-MIP) and the other employing linear problem relaxation supported with column generation and cut generation techniques (referred to as LB-CC). The quality of LBs estimated using these methods is evaluated in Section V.

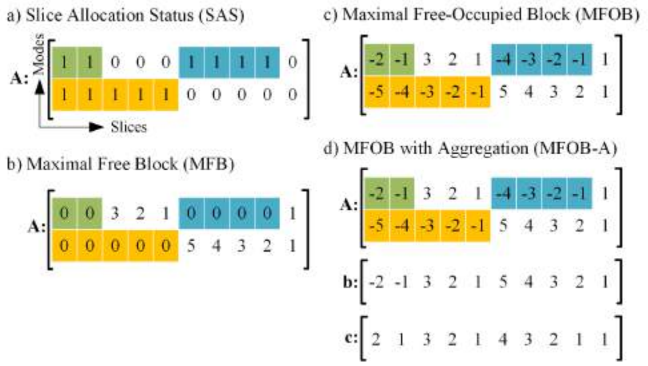

Fig. 1. Different options for representation of availability of spectral-spatial resources in a network link.

A. Relaxing MIP Problem (LB-MIP)

One way to obtain an LB is to solve a simplified MIP problem that does not take the spectrum continuity constraints into account. Such approach has been shown to be effective in EONs [14], [15]. The corresponding problem for an SS-FON with spatial mode conversion can be formulated as follows:

$$\text{minimize } z^{lb} \tag{2a}$$

$$\sum_{p\in\mathcal{P}(d)} x_{dp} = 1 \qquad d \in \mathcal{D} \tag{2b}$$

$$\sum_{m\in\mathcal{M}} x_{dpem} = x_{dp} \qquad d \in \mathcal{D}, p \in \mathcal{P}(d), e \in p \tag{2c}$$

$$\sum_{d\in\mathcal{D}, p\in\mathcal{P}(d): e\in p} n(d,p) \cdot x_{dpem} \leq z^{lb} \quad e \in \mathcal{E}, m \in \mathcal{M}, \tag{2d}$$

where x_{dp} is a binary variable that indicates if path p is used to realize demand d, x_{dpem} is a binary variable that indicates if spatial mode m is used to realize demand d in link e belonging to path p, z^{lb} expresses the (integer) number of slices required in the most utilized mode in a network link, and $n(d, p)$ is the number of slices requested by demand d on path p.

B. Solving LP with Column Generation and Cuts (LB-CC)

Solving MIP problem (2) may be difficult as it contains integer variables, which involves the use of a branch-and-bound algorithm. As an alternative way for estimating LBs, we can consider solving a linear relaxation of problem (1) (referred to as LP). Note that even the LP problem may be challenging if the number of problem variables and constraints is large. Therefore, to solve LP, we employ a column generation (CG) approach, which was shown to be effective in EONs [15], [19].

In CG, the LP problem is initiated with a limited set of problem variables (columns) and additional variables are iteratively generated and included into LP. Since in large problems most columns are irrelevant for the problem (their corresponding variables equal zero in any optimal solution), the processing complexity can be decreased by excluding these columns from the formulation. Note that an unalterable (possibly complete) set of columns is included into the problem when it is solved by an LP solver using a standard method.

The considered LP problem is initiated with a set of allowable lightpaths $\mathcal{L}$ that represents a feasible RSSA solution (found using the greedy algorithm described in Section III). This set is iteratively extended with new lightpaths that are provided by CG. A key element of CG is to formulate and solve a pricing problem (PP). For the RSSA problem in Section II, PP is defined as a problem of finding, for each demand $d \in \mathcal{D}$, a new lightpath l for which its so-called *reduced cost* is positive (and the largest). The reduced cost of primal variable x_{dl} is calculated as $\lambda_d - \sum_{e \in \mathcal{E}(l)}(\sum_{s \in \mathcal{S}(l)} \pi_{es})$, where $\mathcal{E}(l)$ and $\mathcal{S}(l)$ denote, respectively, the set of links and the set of slices used by lightpath l, and λ and π are the vectors representing an optimal dual solution obtained for the current LP. When found, variables x_{dl} representing new lightpaths are included into LP and the resulting LP problem is solved again (in next iteration). Note that after solving LP, the optimal values of dual variables λ_d and π_{es} are obtained directly from the LP solver. Finally, if no such a lightpath exists for all demands, the CG procedure terminates. For more details on CG the reader is referred to [19].

Eventually, similarly as in [15], it is worth to strengthen the LP with the following valid equalities (cuts): $y_s = 1$, $s \in \{1, 2, \ldots, \lceil z^{lb} \rceil\}$, where z^{lb} is the optimal objective value of LP after solving it with CG. Indeed, z is integer in (1) and, therefore, $z \geq \lceil z^{lb} \rceil$ holds. After adding cuts, the resulting LP problem is solved again using CG. This loop (adding cuts and solving LP with CG) is repeated until z^{lb} is an integer value. In this case, rounding it up and setting $y_s = 1$ for $s \in \{1, 2, \ldots, \lceil z^{lb} \rceil\}$ is worthless since it does not have impact on the solution of LP, and the estimation of LB is terminated.

V. Numerical Results

In this section, we evaluate the proposed RSSA optimization procedures in a European network of 28 nodes and 82 links (EURO28) (shown in the bottom-right corner of Fig. 2). We assume the flexgrid of 12.5 GHz granularity and the number of spatial modes $\mathcal{M} \in \{7, 12\}$. The transmission is realized using spectral SChs and polarization division multiplexing. A spectral SCh consists of a number of optical carriers (OCs), each OC occupying 37.5 GHz, and a guard-band of 12.5 GHz. For OCs, we consider four modulation formats: BPSK, QPSK, 8QAM, and 16QAM, of the transmission reach 6300, 3500, 1200, and 600 km [20], and the carried bit-rate 50, 100, 150, and 200 Gbit/s per OC, respectively. We consider that the OCs forming an SCh use the same modulation format. To generate routing paths, we apply a k-shortest path algorithm (assuming physical path lengths) with $k = 10$ paths per demand, and we exclude the paths of length exceeding the maximum transmission reach. Traffic demands have randomly generated end nodes and bit-rates between 50 Gbit/s and 1 Tbit/s, with 50 Gbit/s granularity. All the results are obtained and averaged over 10 randomly generated demand sets.

Numerical experiments are performed on a dual-processor 2.2 GHz 10-core Xeon-class machine (40 logical cores in total) with 128 GB RAM. The performance of SA depends on its parameters, which are: cooling rate and initial temperature coefficient. We apply the same values of these parameters for the studied EON28 network as in [17], namely, the cooling rate is 0.99 and the initial temperature coefficient is 0.05.

We begin with evaluating the LB estimation procedures presented in Section IV, i.e., LB-MIP (with 1-hour run-time limit) and LB-CC. In addition, we include the results of LP relaxation (solved using CG without cuts). To solve the MIP and LP models, we use CPLEX v.12.6.3 [21] (run in a parallel mode and with default settings). In Table I, we show the obtained LB values (z^{lb}) and processing times (T) for $|\mathcal{M}| \in \{7, 12\}$ and $|\mathcal{D}| \in \{200, 400\}$. We can see that LB-CC offers the best performance in terms of z^{lb} (the highest values) and is much faster than LB-MIP. Solving LP (using CG) can be very fast, however, the obtained LBs are of very low quality and it holds also for other, not reported here values of $|\mathcal{M}|$. Therefore, for the following analysis of optimality gaps of the SA-GLA heuristic, we select LB-CC.

TABLE I
Comparison of estimated lower bounds; T in seconds.

		LB-MIP		LB-CC		LP	
$\lvert\mathcal{M}\rvert$	$\lvert\mathcal{D}\rvert$	z^{lb}	T	z^{lb}	T	z^{lb}	T
7	200	58.2	1137	58.4	64	23.9	1
	400	104.2	3600	104.2	482	49.0	3.6
12	200	57.4	40.6	57.4	7.2	14.0	0.6
	400	61.5	2911	62.5	141	28.6	1.3

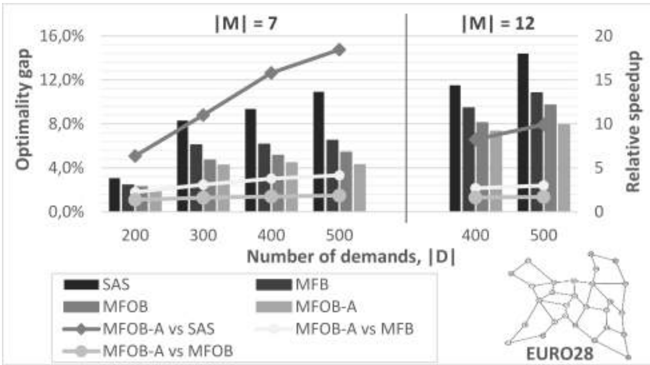

Fig. 2. Comparison of spectrum search procedures in serial SA-GLA with 300 sec. run-time limit, and relative speedup of MFOB-A (vs other options).

Next, we compare the spectrum search procedures presented in Section III run with a serial version of SA-GLA (i.e., 1 thread) with a 300 sec. run-time limit (the short time is due to the large number of executed experiments). In Fig. 2, we can see that the lowest optimality gap, defined as a relative difference between the objective values of LB and heuristic solutions, is obtained with MFOB-A, and the difference between MFOB-A and SAS is of some percents. The optimality gaps increase with the number of demands ($|\mathcal{D}|$) and spatial modes ($|\mathcal{M}|$), which is a result of the complexity of solving larger problem instances. In Fig. 2, we show also a relative speedup of MFOB-A with respect to other spectrum-search options, defined as a ratio of the run-times of a single iteration of the SA-GLA using MFOB-A and the SA-GLA using other option. We can see the the highest speedup of MFOB-A is with respect to SAS (up to 18 times for $|\mathcal{M}| = 7$ and $|\mathcal{D}| = 500$), and it increases with $|\mathcal{D}|$, but decreases with $|\mathcal{M}|$. This gain results from the much faster scanning of spectrum in MFOB-A than in other approaches since MFOB-A skips from processing the entire blocks of

TABLE II
PERFORMANCE OF THE SA-GLA HEURISTIC UNDER DIFFERENT ALGORITHM SETTINGS FOR $|\mathcal{M}| = 7$ AND $|\mathcal{D}| \in \{200, 300, 400\}$.

#	Search	Threads	Run-time	$\|\mathcal{D}\| = 200$		$\|\mathcal{D}\| = 300$		$\|\mathcal{D}\| = 400$	
				z	Δ	z	Δ	z	Δ
(0)	(initial RSSA solution)			89.9	34.4%	114.7	31.5%	140.9	25.6%
(1)	MFOB-A	1	1000 sec.	59.1	1.2%	80.7	3.3%	108	3.5%
(2)	MFOB-A	40	1000 sec.	58.8	0.7%	79.7	2.0%	107.2	2.8%
(3)	SAS	1	1 hour	59.4	1.8%	81.5	4.2%	109.2	4.6%
(4)	MFOB-A	40	1 hour	58.8	0.7%	79.2	1.4%	106.4	2.1%

spectrum that are not suitable for allocation of a demand (as explained in Section III in more details). Consequently, much more iterations of SA-GLA can be performed within the given 300 sec. run-time limit and, thus, better performance results are achieved with MFOB-A than with other approaches.

In Table II, we present performance results of SA-GLA, namely, the objective value of best solution (z) and its optimality gap (Δ), obtained for $|\mathcal{M}| = 7$ and $|\mathcal{D}| \in \{200, 300, 400\}$. Here, our main goal is to compare different aspects of the SA-GLA algorithm and, therefore, the presented results have been obtained for different appropriately selected algorithm settings (indexed from (1) to (4)). For reference, we also include adequate values of initial solutions that initialize the SA-GLA algorithm (under index (0)). First, when comparing (0) with (1)-(4), we can see that SA-GLA is capable of significantly improving the initial solutions. Next, a direct comparison of settings (1) and (2), in which the same spectrum search procedure is applied (i.e, MFOB-A) but different number of threads is assumed, shows that application of parallel processing improves the SA-GLA performance. For instance, the difference in Δ is 1.3% for $|\mathcal{M}| = 300$ if SA-GLA is run with 40 threads instead of 1. Note that the efficiency of parallel processing can be further increased if more sophisticated techniques are applied (e.g., such as exchanging information between threads). Settings (3) and (4) represent, respectively, a baseline version of SA-GLA (run with the SAS approach and 1 thread) and the fully enhanced version of the algorithm. The comparison of (3) and (4) shows that the overall improvement in Δ that comes from both the use of MFOB-A and algorithm parallelization is between about $1\% - 3\%$ (depending on $|\mathcal{D}|$) after 1 hour of SA-GLA performance. Eventually, a comparison of settings (2) and (4), which differ in the considered run-time limit (i.e., 1000 seconds vs 1 hour), shows that the increase of the algorithm processing time may allow SA-GLA to generate better quality solutions.

VI. CONCLUSIONS

We have focused on optimization of RSSA in SS-FONs with spatial mode conversion. We have proposed an efficient optimization algorithm as well as effective procedures for estimating solution lower bounds. We have shown that application of parallel processing and use of dedicated data structures can significantly speed up the search for optimal RSSA solutions. The quality of obtained solutions is high (the optimality gaps reach about $1\% - 2\%$ and below), which is a good result taking into account the size of optimized network instances (in terms of both the number of spatial modes, demands, and network dimension). In future works, we will aim at improving the efficiency of parallel algorithm and will address other SS-FON scenarios including the network without lane changes.

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